Menu Costs and Phillips Curves

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Outline

Motivation: modeling price stickiness

Evidence of nominal rigidity in the form of price stickiness is key to the claim of monetary non-neutrality

Calvo pricing mechanism: firms can change their price at each period with probability $1 - \lambda$

Motivation: price stickiness with menu cost

Golosov and Lucas (2007) presents a model of a monetary economy with menu cost, a firm-specific productivity shock, and an aggregate level money supply shock

Motivation: price stickiness with menu cost

Golosov and Lucas (2007) presents a model of a monetary economy with menu cost, a firm-specific productivity shock, and an aggregate level money supply shock

- at period t=0 and with an initial price p, each firm seeks to maximise expected total profit across all future periods
- the firm faces
  - price p as an inherited price
  - prevailing wage
  -
- the firm chooses:
  - when to change price
  - the new optimal price

Motivation: price stickiness with menu cost

Main objective:

- observing the frequency of price change and output response subject to
  - different deterministic parameter values
  - impulse shocks
- observing impact of impulse shocks on
  - the distribution of price change decisions across firms
  - employment and output

Intuition:

- there is a range of inaction: only firms with prices that are most out-of-line given new shocks will change their price
- When nominal inflation is high, a greater share of firms change their price in each given period to keep up with nominal wage increase
- When menu cost is low, a greater share of firms change their price in each given period

Deriving Monetary Equilibrium Equations - Setup

We will use a New Keynesian framework very similar to the one we already know, with a few modifications. The economy is subject to **stochastic monetary shocks and firm-specific supply shocks.**

Log money supply follows a **Brownian motion:**

$$dlog(m_t) = \mu dt + \sigma_m dZ_m \tag{1}$$

What is a Brownian Motion?

A Brownian Motion is a way to model random walks. μ is a drift parameter indicating the deterministic, expected incremental change over time. Z_m is the random, Brownian motion term and σ_m a volatility parameter.

Deriving Monetary Equilibrium Equations - Setup

Technology shocks are independent across firms and follow a mean-reverting process:

$$dlog(v_t) = -\eta log(v_t)dt + \sigma_v dZ_v, \quad \eta > 0 \tag{2}$$

In equilibrium, **nominal wages** also follow a Brownian Motion:

$$dlog(w_t) = \mu dt + \sigma_m dZ_m \tag{3}$$

Deriving Monetary Equilibrium Equations - Setup

Claims to the monetary unit are traded in a capital market, where

$$E\left[\int_0^\infty Q_t Y_t dt\right] \tag{4}$$

reflects the present value of a dollar earning stream at time 0.

Thus, the **state at time t** is determined by:

- m_t and w_t
- The joint distribution $\phi_t(p, v)$: At time t, the situation of an individual firm depends also on the price p that it carries into t from earlier dates and its idiosyncratic productivity shock. Thus, **each seller is characterized by a pair** (p_t, v_t)**, distributed according to the measure** $\phi_t(p, v)$.

Deriving Monetary Equilibrium Equations - Households

Households **choose a consumption strategy, money holdings** m_t **and a labor supply** l_t**.** By Dixit-Stiglitz aggregation, consumption is given by:

$$c_t = \left[\int C_t(p)^{1-(1/\epsilon)} \phi_t(dp, dv)\right]^{\epsilon/(\epsilon-1)} \tag{5}$$

Households maximize

$$E\left[\int_0^\infty e^{-\rho t}\left[\frac{1}{1-\gamma}c_t^{1-\gamma} - \alpha l_t + \log\left(\frac{\hat{m}_t}{P_t}\right)\right] dt\right] \tag{6}$$

subject to

$$E\left[\int_0^\infty Q_t\left[\int pC_t(p)\phi_t(dp, dv) + R_t\hat{m}_t - W_t l_t - \Pi_t\right] dt\right] \leq m_0 \tag{7}$$

Deriving Monetary Equilibrium Equations - Household FOCs

For money holdings:

$$e^{-\rho t} \frac{1}{m_t} = \lambda Q_t R_t \tag{7}$$

For consumption choices and labor supply:

$$e^{-\rho t} c_t^{-\gamma} c_t^{1/\epsilon} C_t(p)^{-1/\epsilon} = \lambda Q_t p \tag{8}$$

$$e^{-\rho t} \alpha = \lambda Q_t w_t \tag{9}$$

Deriving Monetary Equilibrium Equations - Household FOCs

There is an equilibrium in which the nominal rate is constant at the level

$$R_t = R = \rho + \mu \tag{10}$$

In such an equilibrium, (7), (9), and (10) imply

$$w_t = \alpha R m_t \tag{11}$$

Deriving Monetary Equilibrium Equations - Firms

At a current price level p, a firm profit is

$$C_t(p)\left(p - \frac{w_t}{v_t}\right) \tag{12}$$

At any price $q \neq p$, profits are

$$C_t(q)\left(q - \frac{w_t}{v_t}\right) - kw_t \tag{13}$$

where k is the hours of labor needed to change price - the **menu cost.**

Deriving Monetary Equilibrium Equations - Firms

This firm chooses a shock-contingent repricing time T and a shock-contingent price q to be chosen at t+T to solve the following **Bellman equation**:

$$\varphi\left(p, v, w, \phi_t\right) = \max_T E_t \left[\int_t^{t+T} Q_s C_s(p) \left(p - \frac{w_s}{v_s} \right) ds \right.$$
$$\left. + Q_T \cdot \max_q \left[\varphi\left(q, v_{t+T}, w_{t+T}, \phi_{t+T}\right) - k w_{t+T} \right] \right] \tag{14}$$

What is a Bellman equation?

$$V(state_t) = \max_{control_t} \{u(state_t, control_t) + \beta V(state_{t+1})\}$$

Instead of solving the optimal sequence $\{control_t\}_{t=0}^{T}$, it looks for the time-invariant value function and policy function $control_t = g(state_t)$ that solve the dynamic problem.

Deriving Monetary Equilibrium Equations - Firms

The demand function for each good is:

$$C_t(p) = c_t^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_t}\right)^{-\epsilon} \tag{15}$$

Applying the natural normalization $Q_0 = 1$ to (9), we obtain:

$$Q_t = e^{-\rho t} \frac{w_0}{w_t} \tag{16}$$

Then we can express the Bellman as:

$$\begin{aligned} \varphi\left(p, v, w, \phi_t\right) = \max_T E_t \Bigg[& \int_t^{t+T} e^{-\rho(s-t)} \frac{w}{w_s} c_s^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_s}\right)^{-\epsilon} \left(p - \frac{w_s}{v_s}\right) ds \\ & + e^{-\rho T} \frac{w}{w_T} \cdot \max_q \left[\varphi\left(q, v_{t+T}, w_{t+T}, \phi_{t+T}\right) - k w_{t+T}\right] \Bigg] \end{aligned} \tag{17}$$

Deriving Monetary Equilibrium Equations - Firms

Labor market clearing implies:

$$l_t = \int \frac{C_t(p)}{v} \phi_t(dp, dv) + k\Upsilon_t \tag{18}$$

Let's recap:

- Two kinds of shocks v_t, m_t and a family of measure $\phi_t(p, v)$ shape the states.
- The household chooses goods demand, labor supply, and money-holding strategies according to the states of the world.
- A firm's pricing strategy is now defined as choices of stopping times T and prices q.

Equilibrium under Constant Wage Inflation

(17) is hard to analyze because of the presence of $\phi_t(p, v)$ as a state variable. Either we can provide or construct a law of motion for it, **OR** we can conjecture an equilibrium in which the distributions are all equal to an invariant measure, $\phi(p, v)$

Why $\phi(p, v)$?

$\phi_t(p, v)$ enters (17) only as a determinant of the consumption aggregate. If we use $\phi(p, v)$ instead, we can get a constant corresponding consumption aggregate facing the firms.

Equilibrium under Constant Wage Inflation

Consumption aggregate:

$$c_t = \left[\int \left(\frac{\alpha p}{w_t} \right)^{1-\epsilon} \phi_t(dp, dv) \right]^{1/[\gamma(\epsilon-1)]} \tag{18}$$

Define real price:

$$x = p/w_t \tag{19}$$

Restate (18) as:

$$c_t = \left[\alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_t(dx, dv) \right]^{1/[\gamma(\epsilon-1)]} \tag{20}$$

Here, the process of nominal wage growth is deterministic, thus we can construct a joint distribution $\tilde{\phi}_t$ in the similar form of ϕ_t.

Equilibrium under Constant Wage Inflation

With an invariant measure $\tilde{\phi}$ and a corresponding constant consumption aggregate $\bar{c}$, rewrite the Bellman as:

$$\varphi(p, v, w) = \max_T E\left[\int_0^T e^{-\rho s}\frac{w}{w_s}\bar{c}^{1-\epsilon\gamma}\left(\frac{\alpha p}{w_s}\right)^{-\epsilon}\left(p - \frac{w_s}{v_s}\right)ds \right.$$
$$\left. + e^{-\rho T}\frac{w}{w_T}\cdot\max_q\left[\varphi\left(q, v_T, w_T\right) - kw_T\right]\right] \tag{21}$$

Use x, instead of p/w_t to restate:

$$\frac{1}{w}\varphi(wx, v, w) = \max_T E\left[\int_0^T e^{-\rho s}\bar{c}^{1-\epsilon\gamma}\left(\alpha x_s\right)^{-\epsilon}\left(x_s - \frac{1}{v_s}\right)ds \right.$$
$$\left. + e^{-\rho T}\frac{1}{w_T}\cdot\max_{x'}\left[\varphi\left(w_T x', v_T, w_T\right) - kw_T\right]\right] \tag{22}$$

Equilibrium under Constant Wage Inflation

Finally, we seek a solution to (22) of the form:

$$\varphi(p, v, w) = w\psi(x, v) \tag{23}$$

Where

$$\psi(x, v) = \max_T E\left[\int_0^T e^{-\rho t}\bar{c}^{-1-\epsilon}(\alpha x_t)^{-\epsilon}\left(x_t - \frac{1}{v_t}\right)dt + e^{-\rho T}\cdot\max_{x'}\left[\psi\left(x', v(T)\right) - k\right]\right] \tag{24}$$

Find the value of $\bar{c}$ by solving the fixed-point problem:

$$\bar{c} = \left[\alpha^{1-\epsilon}\int x^{1-\epsilon}\tilde{\phi}_t(dx, dv; \bar{c})\right]^{1/[\gamma(\epsilon-1)]} \tag{25}$$

Equilibrium under Constant Wage Inflation

Now study the Bellman equation, using a discrete-time and state approximation—a Markov chain:

$$\psi(x,v) = \max\left\{\Pi(x,v)\Delta t + e^{-r\Delta t}\sum_{x',v'}\pi\left(x',v' \mid x,v\right)\psi\left(x',v'\right),\right.$$
$$\left.\max_{\xi}\left[\Pi(\xi,v)\Delta t + e^{-r\Delta t}\sum_{x',v'}\pi\left(x',v' \mid \xi,v\right)\psi\left(x',v'\right)\right] - k\right\} \tag{26}$$

Under the assumption that:

$$\Pi(x,v) = \bar{c}^{-1-\epsilon}(\alpha x)^{-\epsilon}\left(x - \frac{1}{v}\right) \tag{27}$$

Equilibrium under Constant Wage Inflation

Define function:

$$\Omega(v) = \max_x [\psi(x, v)] \tag{28}$$

So $\Omega(v)$ is the value the firm would have if it could move costlessly to a new price when the wage is w and the productivity level is v.

$$D(v) = \{x > 0 : \psi(x, v) > \Omega(v) - k\}, \tag{29}$$

So $D(v)$ is the set at which a firm does not pay to re-price, the **Inaction Region**. The policy function for (26) thus can be defined by:

$$f(x, v) = x \text{ if } x \in D(v)$$
$$f(x, v) = g(v) \text{ if } x \notin D(v)$$

Figure 1

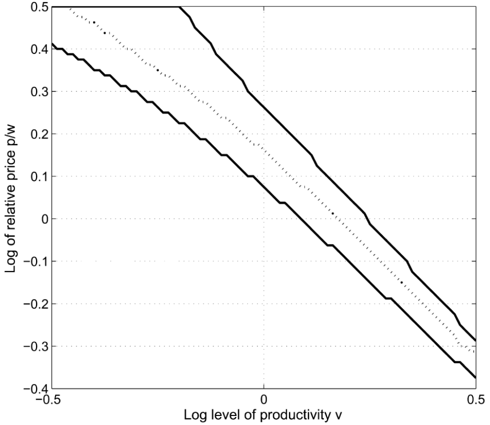


FIG. 1.—Pricing bounds for 0.64 percent quarterly inflation. Solid lines: upper and lower bounds $U(v)$ and $L(v)$. Dotted line: $g(v)$.

Figure 1: How prices react to productivity shocks

Impulse Responses | Results | Results ct'd

Testing the Model

We now have a set of equations characterizing equilibrium. We will show you that the model works remarkably well to predict empirical evidence.

Data (Klenow and Kryvstov, 2005)

- Bureau of Labor Statistics (BLS) survey from the US
- Ca. 80.000 time series on price quotes in 88 locations from 1988 to 1997 (monthly or bimonthly frequency)
- First, we use **real data to calibrate** the model. Then, we use the model to **predict what happens when parameters change.**

- **Results:** Frequency of price changes is
  - Increasing in ${\sigma^2}_v$ (shock variance)
  - Decreasing in k (menu cost)
  - Insensitive to η (rate of mean reversion)

Calibrating the Model

TABLE 1
CALIBRATED PARAMETER VALUES
Baseline Values: $(\eta, \sigma_v^2, k) = (.55, .011, .0025)$

Moment	Data (1)	Model (2)	$\eta = .65$ (3)	$\sigma_v^2 = .015$ (4)	$k = .002$ (5)
Quarterly inflation rate	.0064	.0064	.0064	.0064	.0064
Standard deviation of inflation	.0062	0	0	0	0
Frequency of change	.219	.239	.232	.273	.269
Mean price increase	.095	.097	.094	.104	.092
Standard deviation of new prices	.087	.090	.080	.108	.091

NOTE.—Col. 2 is based on the baseline values. Cols. 3–5 are based on the same values, except for the changes indicated at the head of each column.

Figure 2: Model Calibration

Notice: ${\sigma^2}_m$ is set to zero to model a deterministic inflation trend.

Simulating the Model

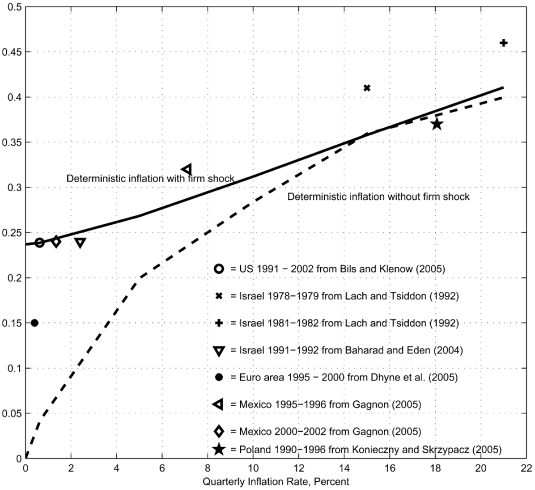


FIG. 3.—Fraction of prices changed each month

Figure 3: Simulations across various inflation rates. What do you see?

Simulating the model

The model predicts data very well - *better* than conventional models that do not include firm-specific shocks.

Specifically, the model works well **in low inflation economies,** where conventional models fail. **Why?**

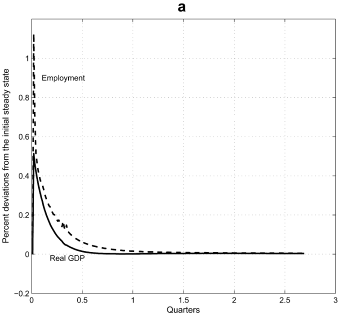
Simulating the model

The model predicts data very well - *better* than conventional models that do not include firm-specific shocks.

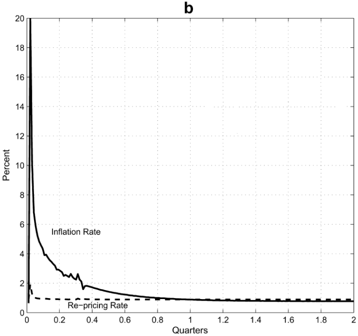
Specifically, the model works well **in low inflation economies,** where conventional models fail. **Why?**

Because in **low inflation environments,** it matters more *who* changes their price - which is exactly what this model captures.

Impulse Response Functions to a Monetary Shock



(a) Response to a transient monetary shock of real GDP and employment



(b) Response to a transient monetary shock of the repricing and inflation rate

Impulse Response Functions to a Monetary Shock

- Suppose the money supply increases by 1,25 percent. This leads to an **unanticipated wage increase**
- Output increases by less than the monetary shock
- Some firms temporarily change their prices, but this effect does not last long
- Effect on real GDP subside fast since firms who did not initially react to aggregate shock reprice due to idiosyncratic shocks

Bottom line: Monetary shocks do not have a long-lasting effect in this setting

Figure 1

Back to Calvo

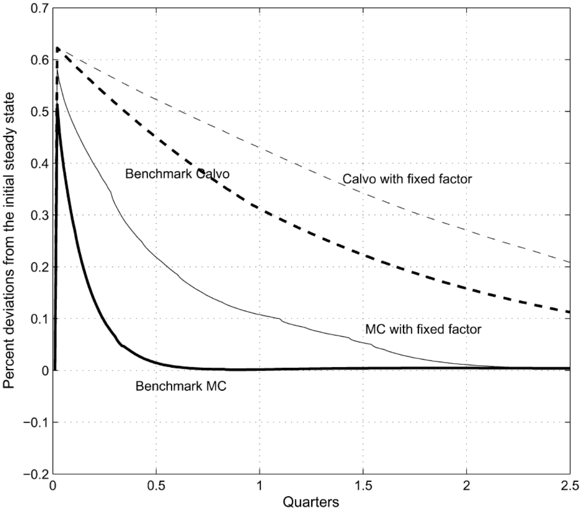


FIG. 5.—Output responses in menu cost and Calvo models

Results Results ct'd

Figure 5: MC vs. Calvo

Output Response to a Monetary Shock

- Under Calvo assumptions, the response of output to a positive monetary shock is **larger and more persistent** compared to the menu cost model
- To understand why, let's compare firm repricing behaviour before (6a) and after (6b) such a shock:

Back to Calvo

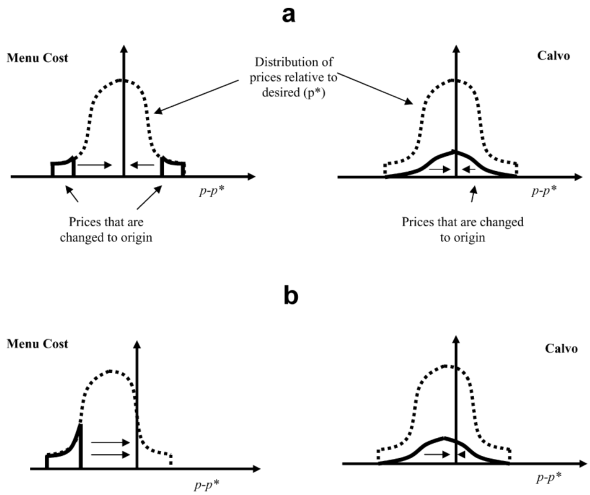


FIG. 6.—Price adjustment in menu cost and Calvo models. *a*, Price adjustment before aggregate shock. *b*, Price adjustment after aggregate shock.

Results

Results ct'd

Figure 6: Price adjustments in Calvo and MC models

Results

Suppose no aggregate shock has occurred yet (Figure 6a):

- Under Calvo assumptions, **price changes are random and smaller on average**
- Under Calvo, price-changing firms are selected randomly, including those close to their desirable price
- Under menu costs,large price adjustments occur when a firm is far off its desirable price
- **The position of a firm in the relative price distribution matters!**

Figure 1 Figure 5 Figure 6

Results

Now consider an aggregate monetary shock:

- During an aggregate shock, a firm will want to raise its price, and the relative price distribution **shifts to the left**
- Firms far to the left will increase prices quickly
- Others will wait since inflation offsets negative v shock
- Under Calvo, the response of prices is much smaller and **takes longer** - impulse responses become more persistent

Overall: Price adjustments per firm are larger and no longer random.

What matters is not so much how many prices are changed, but which prices are changed. Golosov and Lucas (2007)

Figure 1 Figure 5

Discussion

- What if we assumed wage stickiness?

Discussion

- Section 7: Stochastic inflation

Concluding Remarks

References